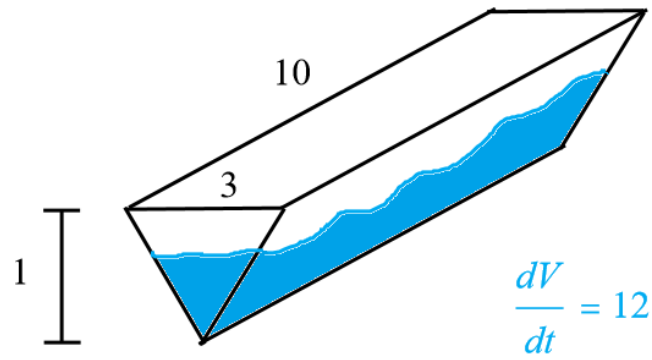
Exercise 26

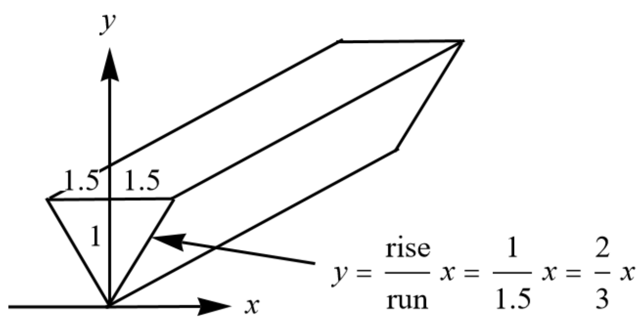
A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?

Solution

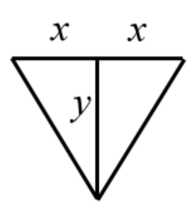
Start by drawing a schematic of the trough at a certain time.



Write an equation for the line representing the triangle's edge.



The aim is to find the volume $V(y)$ that water occupies if it's at a height y. In order to do this, find the area of a cross-section and then multiply it by 10 feet, the thickness.



This is the area of a triangle.

$$A = \frac{1}{2}(2x)y$$

$$= xy$$

Since we want to find dy/dt when $y = 0.5$, eliminate x in favor of y.

$$A = \left(\frac{3y}{2}\right) y$$

$$= \frac{3y^2}{2}$$

Multiply the area by the thickness, 10 feet, to get the volume.

$$V = 10A$$

$$= 10\left(\frac{3y^2}{2}\right)$$

$$= 15y^2$$

Take the derivative of both sides with respect to t by using the chain rule.

$$\frac{d}{dt}(V) = \frac{d}{dt}(15y^2)$$

$$\frac{dV}{dt} = 30y \cdot \frac{dy}{dt}$$

$$12 = 30y\frac{dy}{dt}$$

Solve for dy/dt.

$$\frac{dy}{dt} = \frac{2}{5y}$$

Therefore, the rate that the water level is rising when the water is 6 inches (0.5 feet) deep is

$$\left.\frac{dy}{dt}\right|_{y=0.5} = \frac{2}{5(0.5)} = \frac{4}{5} = 0.8\ \frac{\text{ft}}{\text{min}}.$$